(12)	cB, bcA, bAB, $abcC$, $abBC$, aAC, $acABC$;
(13)	abB, $abcA$, cAB, bcC, $acBC$, aAC, $bABC$;
(14)	acB, bA, $abcAB$, bcC, $abBC$, cAC, $aABC$;
(15)	acB, abA, $bcAB$, bC, $abcBC$, aAC, $cABC$;
(16)	acB, $abcA$, bAB, aC, cBC, $bcAC$, $abABC$;
(17)	bcB, $abcA$, aAB, acC, $abBC$, bAC, $cABC$;
(18)	$abcB$, acA, bAB, bcC, aBC, $abAC$, $cABC$.

But (12) has an element in common with each of the sets (7)—(18). This is also true if we start with (13), ..., or (18) instead of (12). Hence the fifth, sixth, ..., ninth sets must be chosen from (7)—(11). Inversely, no two of the sets (6)—(11) have an element in common, and hence give a solution of the problem. We have now proved that

$$N_3=(5.9.31)8^3(7.6.4)(6.4.2)5!\div 9!=2^{12}.3.5.31).$$

Theorem. *Each of the* $2^{12}.3.5.31$ *solutions of the problem may be derived by a suitable change of notation from the solution given by the nine sets* (6)—(11).

THE UNIVERSITY OF CHICAGO, *June, 1904.*

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

205. Proposed by G. B.M. ZERR, A. M., Ph. D., Parsons, W. Va.

Express in the form of radicals the roots of the equation:

$$x^{15}+15mx^{13}+90m^2x^{11}+275m^3x^9+450m^4x^7+378m^6x^5+140m^6x^3+15m^7x+2r=0.$$

I. Solution by A. H. HOLMES, Brunswick, Maine.

$x^{15}+15mx^{13}+90m^2x^{11}+275m^3x^9+459m^4x^7+378m^5x^5+140m^6x^3+15m^7x+2r=0.$

$\therefore (x^5+5mx^3+5m^2x)+3m^5(x^5+5mx^3+5m^2x)+2r=0.$

Put $x^5+5mx^3+5m^2x=y$.

$\therefore y^3+3m^5y+2r=0.$

By the well known method of solving cubics:

$y_1=[-r+\sqrt{(m^{15}+r^2)}]^{\frac{1}{3}}+[-r-\sqrt{(m^{15}+r^2)}]^{\frac{1}{3}}=t_1+t_2=R_1,$

$y_2=-\frac{1}{2}(t_1+t_2)+\frac{1}{2}\sqrt{-3}(t_1-t_2)=R_2,$

$y_3=-\frac{1}{2}(t_1+t_2)-\frac{1}{2}\sqrt{-3}(t_1-t_2)=R_3,$

$\therefore x^5+5mx^3+5m^2x=R_1$ or R_2 or R_3.

Taking the first of these values,

$$x_1=\left(\frac{R_1}{2}+\sqrt{m^5+\frac{R_1^2}{4}}\right)^{\frac15}+\left(\frac{R_1}{2}-\sqrt{m^5+\frac{R_1^2}{4}}\right)^{\frac15}=s_1+s_2,$$

$x_2=es_1+e^4s_4$; $x_3=e^2s_1+e^3s_4$; $x_4=e^3s_1+e^2s_4$, and $x_5=e^4s_1+es_4$, when e represents the imaginary fifth root of unity.

The other ten values of x can be found by substituting R_2 and R_3 for R_1 above.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $x=y+z$.

$\therefore y^{15}+z^{15}+(yz+m)[15(y^{13}+z^{13})+(105yz+90m)(y''+z'')+(455y^2z^2+715myz+275m^2)(y^9+z^9)+(1365y^3z^3+2925my^2z^2+2025m^2yz+450m^3)(y^7+z^7)+(3003y^4z^4+7722my^3z^3+7128m^2y^2z^2+2772m^3yz+378m^4)(y^5+z^5)+(5005y^5z^5+14300my^4z^4+15400m^2y^3z^3+7700m^3y^2z^2+1750m^4yz+140m^5)(y^3+z^3)+(6435y^6z^6+19305my^5z^5+22275m^2y^4z^4+12375m^3y^3z^3+3375m^4y^2z^2+405m^5yz+15m^6)(y+z)]+2r=0.$

$\therefore y^{15}+z^{15}=-2r$, $yz=-m$. Let $y^{15}=a$, $z^{15}=b$.

$\therefore a+b=-2r$, $ab=-m^{15}$. $\therefore a$ and b are the roots of $t^2+2rt-m^{15}=0$.

Let β=an imaginary fifteenth root of unity and also let $a^{\frac{1}{15}}=c$, $b^{\frac{1}{15}}=d$.

$\therefore$ The roots are $c+d$, $\beta c+\beta^{14}d$, $\beta^{14}c+\beta d$, $\beta^2c+\beta^{13}d$, $\beta^{13}c+\beta^2d$, $\beta^3c+\beta^{12}d$, $\beta^{12}c+\beta^3d$, $\beta^4c+\beta^{11}d$, $\beta^{11}c+\beta^4d$, $\beta^5c+\beta^{10}d$, $\beta^{10}c+\beta^5d$, $\beta^6c+\beta^9d$, $\beta^9c+\beta^6d$, $\beta^7c+\beta^8d$, $\beta^8c+\beta^7d$.

Also solved by J. Scheffer.

206. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

The product of a certain pair of roots of $x^4+ax^3+bx^2+amx+m^2=0$, is equal to the product of the remaining pair.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let x_1, x_2, x_3, x_4 be the roots. Then $x_1+x_2+x_3+x_4=-a$; $x_1x_2x_3+x_1x_2x_4+x_1x_3x_4+x_2x_3x_4=-am$; $x_1x_2x_3x_4=m^2$.

$\therefore m(x_1+x_2+x_3+x_4)=x_1x_2(x_3+x_4)+x_3x_4(x_1+x_2)=-am$.

$\therefore x_1x_2=x_3x_4=m$.

The same is true if $x_1x_3=x_2x_4=m$, $x_1x_4=x_2x_3=m$.

II. Solution by J. SCHEFFER, Kee Mar College, Hagerstown, Md.

From the theory of equations, we have $\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=-am$, or $\alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)=-am$, but $\gamma\delta=\alpha\beta$.

$\therefore \alpha\beta(\alpha+\beta+\gamma+\delta)=-am$. Since $\alpha+\beta+\gamma+\delta=-a$, $-\alpha\beta a=-am$.

$\therefore \alpha\beta=m=\gamma\delta$.